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A Sensitivity Analysis Procedure for Quality Function Deployment (QFD)

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QFD Phase 1

General Steps

1. Determine the customer needs and corresponding technical constraints. Prioritize the importance of the needs.
2. Rate the relationships between the customer needs and the technical constraints ($\longrightarrow$ QFD matrix).

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4. Analyze the result(s).

Example

Initial Solution

CRs	goal τ_y	TC1	TC2	TC3	TC4	TC5	TC6
CR1	0.41			1	1		
CR2	0.28			9	9	9	1
CR3	0.34	3	3			9	
CR4	0.28	9	1	3	9		9
CR5	0.30		3			9	9
CR6	0.44		3	3	1		
CR7	0.43			1	3		
CR8	0.30			9	9		9

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Response–Centric Approach

1. Solve the eigensystem $AA^T y = \lambda y$

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Initial Solution

CRs	goal τ_y	TC1	TC2	TC3	TC4	TC5	TC6	y
CR1	0.41			1	1			0.04
CR2	0.28			9	9	9	1	0.52
CR3	0.34	3	3			9		0.16
CR4	0.28	9	1	3	9		9	0.52
CR5	0.30		3			9	9	0.32
CR6	0.44		3	3	1			0.09
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Optimal Solution

CRs	goal τ_y	TC1	TC2	TC3	TC4	TC5	TC6	y
CR1	0.41			7	6			0.42
CR2	0.28			3	4	3	1	0.29
CR3	0.34	8	7			8		0.34
CR4	0.28	3	1	2	4		2	0.28
CR5	0.30		5			6	8	0.30
CR6	0.44		4	6	5			0.43
CR7	0.43			6	7			0.42
CR8	0.30			3	4		5	0.30
x		0.20	0.32	0.57	0.61	0.30	0.26	0.02

Question

How to proceed

when the convergence gap

is not acceptable small?

Definition & Objective

Definition

A **transfer function** f maps the input parameters $x_1, x_2, \ldots, x_n$ to the output y of a process, mathematically expressed by

$$y = f(x_1, x_2, \ldots, x_n) = f(\boldsymbol{x}).$$

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Objective

The intention of a transfer function f is to predict the (nearly) optimal settings of the controls $\boldsymbol{x}$ in order to achieve a predefined **target** τ_y of the response y.

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A transfer function $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ with $\boldsymbol{y} = f(\boldsymbol{x})$, $\boldsymbol{y} \in \mathbb{R}^m$, and target $\boldsymbol{\tau}_{\boldsymbol{y}} \in \mathbb{R}^m$ is called a **multiple–response transfer function**.

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Example

Assume that $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is defined by a matrix $\boldsymbol{A} \in \mathbb{R}^{m \times n}$. Then

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}, \quad \boldsymbol{x} \in \mathbb{R}^n, \quad \boldsymbol{y} \in \mathbb{R}^m,$$

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is termed a **linear multiple–response transfer function**.

1. The entries a_{ij} of $\boldsymbol{A}$ are the controls of f, and
2. $\boldsymbol{y}$ is (nearly) optimal when convergence gap $\|\boldsymbol{y} - \boldsymbol{\tau_y}\|_2 \leq \varepsilon$.

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5. ascertains interaction effects of the controls.

Types of Procedures

Local Methods

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Global Methods

examine the entire set of controls and evaluate the effect a specified control has on a response while the remaining controls may be altered as well. For instance: **Winding Stairs Method**.

The Winding Stairs Method

Preliminaries

1. Set the convergence gap threshold $\varepsilon > 0$ and the maximum number m of Winding Stairs cycles.
2. Identify the set $\mathcal{F}_{\boldsymbol{A}}$ of feasible entries of matrix $\boldsymbol{A}$ and cluster the elements of $\mathcal{F}_{\boldsymbol{A}}$ row–wise.
3. Calculate an initial response $\boldsymbol{y}$ by solving the eigensystem $\boldsymbol{A}\boldsymbol{A}^T\boldsymbol{y} = \lambda\boldsymbol{y}$.
4. Define the access order of the rows by means of a specific procedure, e.g., sort the components of $|\boldsymbol{y} - \boldsymbol{\tau}_{\boldsymbol{y}}|$ in descending order.
5. Compute $\boldsymbol{x} = \boldsymbol{A}^T\boldsymbol{y}$.

The Winding Stairs Method

Modifying the Controls

Sample	Actual Controls	Response	New Value
$\boldsymbol{x_1}$	$(x_1^{(1)}, x_2^{(1)}, x_3^{(1)}, \ldots, x_n^{(1)})$	$f(\boldsymbol{x_1})$	
$\boldsymbol{x_2}$	$(x_1^{(1)}, \boldsymbol{x_2^{(2)}}, x_3^{(1)}, \ldots, x_n^{(1)})$	$f(\boldsymbol{x_2})$	$\boldsymbol{x_2^{(2)}} = x_2^{(1)} + \Delta x_2^{(1)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\boldsymbol{x_n}$	$(x_1^{(1)}, x_2^{(2)}, x_3^{(2)}, \ldots, \boldsymbol{x_n^{(2)}})$	$f(\boldsymbol{x_n})$	$\boldsymbol{x_n^{(2)}} = x_n^{(1)} + \Delta x_n^{(1)}$
$\boldsymbol{x_{n+1}}$	$(\boldsymbol{x_1^{(2)}}, x_2^{(2)}, x_3^{(2)}, \ldots, x_n^{(2)})$	$f(\boldsymbol{x_{n+1}})$	$\boldsymbol{x_1^{(2)}} = x_1^{(1)} + \Delta x_1^{(1)}$
$\boldsymbol{x_{n+2}}$	$(x_1^{(2)}, \boldsymbol{x_2^{(3)}}, x_3^{(2)}, \ldots, x_n^{(2)})$	$f(\boldsymbol{x_{n+2}})$	$\boldsymbol{x_2^{(3)}} = x_2^{(2)} + \Delta x_2^{(2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\boldsymbol{x_{2n}}$	$(x_1^{(2)}, x_2^{(3)}, x_3^{(3)}, \ldots, \boldsymbol{x_n^{(3)}})$	$f(\boldsymbol{x_{2n}})$	$\boldsymbol{x_n^{(3)}} = x_n^{(2)} + \Delta x_n^{(2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\boldsymbol{x_{mn}}$	$\left(x_1^{(m)}, x_2^{(m+1)}, x_3^{(m+1)}, \ldots, \boldsymbol{x_n^{(m+1)}}\right)$	$f(\boldsymbol{x_{mn}})$	$\boldsymbol{x_n^{(m+1)}} = x_n^{(m)} + \Delta x_n^{(m)}$

The Winding Stairs Method

Winding Stairs Matrix

$$\begin{pmatrix} f(x_1) & f(x_2) & f(x_3) & \cdots & f(x_n) \\ f(x_{n+1}) & f(x_{n+2}) & f(x_{n+3}) & \cdots & f(x_{2n}) \\ f(x_{2n+1}) & f(x_{2n+2}) & f(x_{2n+3}) & \cdots & f(x_{3n}) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ f(x_{(m-1)n+1}) & f(x_{(m-1)n+2}) & f(x_{(m-1)n+3}) & \cdots & f(x_{mn}) \end{pmatrix}$$

WS Matrix: Example

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0.7731	0.7715	0.7688	0.7649	0.7613	0.7584	0.7491	0.7381	0.7284	0.7170	0.7074	0.6962	0.6926
2	0.6640	0.6630	0.6603	0.6559	0.6503	0.6448	0.6347	0.6230	0.6120	0.5990	0.5879	0.5753	0.5717
3	0.5307	0.5299	0.5299	0.5252	0.5181	0.5104	0.4990	0.4873	0.4733	0.4582	0.4446	0.4306	0.4251
4	0.3700	0.3684	0.3684	0.3612	0.3544	0.3469	0.3395	0.3347	0.3194	0.3054	0.2936	0.2844	0.2705
5	0.2002	0.2002	0.2002	0.1989	0.1978	0.1940	0.1940	0.1940	0.1771	0.1668	0.1640	0.1640	0.1479
6	0.0889	0.0889	0.0876	0.0876	0.0842	0.0735	0.0735	0.0735	0.0717	0.0717	0.0717	0.0717	0.0642
7	0.0443	0.0443	0.0443	0.0443	0.0432	0.0415	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386
8	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221
9	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221
10	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221
11	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221

	14	15	16	17	18	19	20	21	22	23	24	25
1	0.6878	0.6822	0.6797	0.6769	0.6769	0.6749	0.6749	0.6744	0.6715	0.6673	0.6666	0.6666
2	0.5670	0.5602	0.5586	0.5554	0.5554	0.5551	0.5517	0.5484	0.5451	0.5399	0.5349	0.5349
3	0.4196	0.4124	0.4124	0.4114	0.4090	0.4086	0.4056	0.4037	0.3947	0.3839	0.3755	0.3755
4	0.2587	0.2537	0.2499	0.2461	0.2461	0.2455	0.2436	0.2436	0.2288	0.2159	0.2044	0.2044
5	0.1403	0.1403	0.1375	0.1375	0.1375	0.1375	0.1375	0.1356	0.1188	0.1101	0.0937	0.0937
6	0.0642	0.0603	0.0603	0.0603	0.0603	0.0603	0.0603	0.0594	0.0523	0.0523	0.0463	0.0463
7	0.0386	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221
8	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221
9	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221
10	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221
11	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221	0.0221

Measuring Effects

Law of Total Variance

$$\begin{aligned} V(Y) &= V_{X_i}\left(E_{\boldsymbol{X}_{\sim i}}(Y|X_i)\right) + E_{X_i}\left(V_{\boldsymbol{X}_{\sim i}}(Y|X_i)\right) \\ &= \text{expected reduction in } V(Y) \text{ when } X_i \text{ could be fixed} \\ &\quad + \text{expected residual variance w.r.t. } \boldsymbol{X}_{\sim i} \text{ with fixed } X_i \end{aligned}$$

Main Effect Index

$$S_i = \frac{V_{X_i}\left(E_{\boldsymbol{X}_{\sim i}}(Y|X_i)\right)}{V(Y)}, \qquad 1 \le i \le n$$

Total Effect Index

$$S_{T_i} = \frac{E_{\boldsymbol{X}_{\sim i}}\left(V_{X_i}(Y|\boldsymbol{X}_{\sim i})\right)}{V(Y)} = 1 - \frac{V_{\boldsymbol{X}_{\sim i}}\left(E_{X_i}(Y|\boldsymbol{X}_{\sim i})\right)}{V(Y)}, \quad 1 \le i \le n$$

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